The Cosmological Constant Problem and Quantum Spacetime Reference Frame

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We generalize the idea of quantum clock time to a quantum spacetime reference frame via a physical realization of a reference system by quantum rulers and clocks. Omitting the internal degrees of freedom (such as spins) of the physical rulers and clocks, only considering their metric properties, the spacetime reference frame is described by a bosonic non-linear sigma model. We study the quantum behavior of the system under approximations, and obtain (1) a cosmological constant valued $(2/\pi)\rho_{c0}$ (ρ_{c0} is the critical density measured at near current epoch) which is very close to current observations; (2) an Einstein-Hilbert term in the quantum effective action; (3) the ratio of variance to mean-squared of spacetime interval tends to a universal constant $2/\pi$ in the infrared region. This effect is testable by observing a linear dependence between the inherent quantum variance and mean-squared of the redshifts from cosmic distant spectral lines. The proportionality is expected to be the observed percentage of the dark energy. We also generalize the equivalence principle to be valid for all quantum phenomenon.

I. INTRODUCTION

Reference frame is one of the most fundamental notions in physics. When a measurement in physics is performed or described, a reference frame has always been explicitly or implicitly used. As ordinarily formulated, reference frame idealizationally uses the rulers and clocks to label the spacetime for simplicity, which have well-defined values of coordinates and are considered most perfect, absolute, classical, and external. This fundamentally classical notion of reference frame has been using in almost all area of physics including today's textbook quantum physics, although quantum mechanics has been discovered for a century. The quantum mechanics tells us that all measuring devices are subject to some level of quantum fluctuations, certainly applying to the rulers and clocks, namely the spacetime. Such idealizational treatment works well in quantum mechanics and quantum field theory when the equations of them cast in terms of the variables that are really measured by physical rulers and clocks in ordinary laboratory. This is, to a large extent, due to the fact that gravitational effects are not seriously taken into account in almost laboratory experiments. Since according to the standard theory of gravity, the general relativity, the spacetime is dynamical and relational. It is as expected, when the quantum mechanics is applied to the cosmology which is gravity dominated, severe difficulty arises: the cosmological constant problem, see for instance [1–3] and references therein.

Along this line of thinking, treating the concept of a reference frame in quantum theory may be the key to the cosmological constant problem. The earlier publications [4, 5] have proposed a possible solution by replacing the idealized parameter time in textbook quantum theory by a quantum dynamical variable playing the role of a physical clock time. The papers obtain a cosmological constant having not only a correct order but also a percentage $\Omega_\Lambda = 2/\pi \approx 0.64$ which is very close to current observational value [6]. However, such solution can only be regarded as incomplete, since although time has been treated quantum mechanically, in those papers, the spatial coordinates are still treated as classical external parameters free from quantum fluctuations. The space and time measurements are closely related to each other and must be treated on an equal footing. In this work, we shall generalize the discussion of quantum clock time in refs.[4, 5] to a more general framework: quantum spacetime reference frame. It is a years old idea (see for instance [7–10] and references therein), but to the best of our knowledge, there is no literature or discussion yet connecting such idea to the cosmological constant problem, the goal of the paper is to show their profound relation. We take the natural unit $\hbar = c = 1$ for convention in the paper.

II. QUANTUM SYSTEM RELATIVE TO A QUANTUM REFERENCE FRAME

When rulers, clocks and other measuring devices that a to-be-studied physical system is relative to are inherently quantum mechanical, they can not just be ignored from a complete quantum description. In a quantum treatment, a complete set of states of a to-be-studied physical system (denoted by P) plus a quantum reference frame system

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(denoted by R) are described by the Hilbert space $\mathcal{H}$ being a direct product of both Hilbert spaces,

$$\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_R. \tag{1}$$

However, it does not mean that the state vector of the whole system is simply a direct product of vector in each Hilbert space. In most cases, it is an entangled state. The reason is that, in a first step of performing a measurement, instrument calibration must be carried out. A good experimental calibration establishes a one-to-one correlation between a state of the to-be-studied physical system $|P\rangle_i$ and a state of the measuring device $|R\rangle_j$. If the calibration is well done, such step introduces a complete basis $|P\rangle_i \otimes |R\rangle_j$ to expand a state $|\Psi\rangle$ in the whole Hilbert space $\mathcal{H}$,

$$|\Psi\rangle = \sum_{i,j} \alpha_{ij} |P\rangle_i \otimes |R\rangle_j \,. \tag{2}$$

In the second step, physicist let these two systems evolve independently and observe the output of the measuring device. The physics of the individual measuring device is assumed well understood, knowing everything about the individual Hilbert space $\mathcal{H}_R$, and a state in it can be expanded by a complete basis $|R\rangle = \sum_j \beta_j |R\rangle_j$. Then physicist uses the information of calibration Eq.(2) to judge what is the state of P when the output of the measuring device being in one of the state $|R\rangle_j$. The expansion coefficient α_{ij} measures the amplitude of P being in the state $|P\rangle_i$, and meanwhile, the measuring device being in the state $|R\rangle_j$. The $|\alpha_{ij}|^2$ measures the joint probability that each of P, R is in the state $|P\rangle_i$, $|R\rangle_j$.

A. Relational Interpretation

The entangled state has a general property that the amplitude α_{ij} can not be factorized into a product of each amplitude of $|P\rangle$, $|R\rangle$, namely, for all amplitudes γ_i, β_j, defined by $|P\rangle = \sum_i \gamma_i |P\rangle_i$ and $|R\rangle = \sum_j \beta_j |R\rangle_j$, we always have $\alpha_{ij} \neq \gamma_i \beta_j$. This property has important physical implications.

A physical meaningful probability in a measurement is the probability of state $|P\rangle_i$ given the condition that the measuring device is observed in state $|R\rangle_j$. The conditional probability $\mathrm{Pro}(P_i|R_j)$ is defined by

$$\mathrm{Pro}(P_i|R_j) \equiv \frac{\mathrm{Pro}(P_i \cap R_j)}{\mathrm{Pro}(R_j)} = \frac{|\alpha_{ij}|^2}{|\beta_j|^2}, \tag{3}$$

in which the $\mathrm{Pro}(P_i \cap R_j)$ stands for the joint probability $|\alpha_{ij}|^2$. The $\mathrm{Pro}(R_j)$, which is given by $|\beta_j|^2$, represents the probability of state $|R\rangle_j$ when the measuring device independently evolves in the second step. Since $\alpha_{ij} \neq \gamma_i \beta_j$, the conditional probability that P is in $|P\rangle_i$ given R being in $|R\rangle_j$ does not equal to the probability that P is individually in state $|P\rangle_i$, i.e. $\mathrm{Pro}(P_i|R_j) \neq \mathrm{Pro}(P_i)$. In this sense, the occurrence of $|P\rangle_i$ is affected by the probability distribution of $|R\rangle_j$. The state $|P\rangle_i$ of the to-be-studied system has no individual absolute meaning, it makes sense only relative to the state $|R\rangle_j$ of the measuring device as a reference [11, 12], namely, the state is relational [13, 14].

The entangled state is purely a quantum state, having no classical correspondence. Only when the measuring devices are treated semi-classically, i.e. a delta distribution for $|R\rangle_j$, the sign of inequality becomes an equal sign, in this limit it recovers the absolute probability interpretation of the textbook quantum mechanics. When in general the distribution for $|R\rangle_j$ has a finite width due to quantum fluctuation, one can prove that it is equivalent to a textbook wavefunction of P with a smeared coordinate [15].

In this paper, we argue that when a quantum theory is completely formulated in terms of the states entangling a to-be-studied system with measuring devices, and interpreted in a relational manner, the quantum theory is able to accommodate the spirit of relativity, leading to a consistent theory of quantum spacetime reference frame.

B. A Mathematical Model of Spacetime Reference Frame

To take a further look into the Hilbert space of the reference frame $\mathcal{H}_R$, let us considering an operational setup to realized the physical rulers and clocks at quantum level. Suppose we have certain free particles, moving in d-dimensional space x_a, $(a = 0, 1, 2, d-1)$. Omitting the internal degrees of freedom of them such as spin, let $X_\mu(x)$, $(\mu = 0, 1, 2, ..., D-1)$ represent scalar free fields having dimension $[\mathrm{mass}]^{-1}$. As ordinarily described, the space x

can be interpreted as the coordinates according to the walls and clock of a laboratory. Assuming that these identical scalar free fields evolve independently in the laboratory, the resulting action is separable,

$$S_R[X] = \sum_\mu S_{sf}[X_\mu], \tag{4}$$

where each action is formally identical and involves only one single field X_μ. The action for each scalar free field formulated with respect to the laboratory is the standard free field action

$$S_{sf}[X_\mu] = \frac{\lambda}{2}\int d^4x \sum_a \left(\frac{\partial X_\mu}{\partial x_a}\right)^2, \tag{5}$$

in which λ is a constant having dimension $[\text{mass}]^d$ and related to the mass scale (with respect to the frame x) of the identical scalar particles, and since we have used the laboratory wall interpretation to x_a so here $d = 4$.

In a range of laboratory scale, the walls and clock of the laboratory can be used to orient, align and order the beams of these scalar particles at high precision, in this sense, these identical quantum scalar free fields can be interpreted as physical rulers and clock at quantum level. The identical scalar fields oriented as $X_{1,2,3} = X, Y, Z$ can be aligned with a reference to the x, y, z-directions of the walls of the laboratory respectively. One could visualize them as local quantum vibrations or oscillations placed on the lattice of x, y, z, these identical particles can be seen as rulers since distances are able be measured by counting the phase changes of the local vibrations if events trigger the counting. In this sense they play the roles of state-triggers labeling where the event happens. For the same consideration, a scalar field denoted as $X_0 = T$ on the lattice can be used to play the role of a small pendulum clock labeling the causal order of the events, i.e. when the event happens. As a result, an event denoted by state $|P\rangle_i$ is correlated to a particular vibrations configuration or a state of the identical particles system it triggers

$$|R\rangle_j = |X_0, X_1, X_2, X_3\rangle, \tag{6}$$

according to the entanglement Eq.(2). So we could say that a state of the identical particles as a standard reference system labels where and when a quantum event happens, playing the role of a spacetime reference frame.

For the boson statistics of the four identical particles, the interchange between them does not change the state $|R\rangle_j$. In one case, because the role of the vibrations on the lattice as ruler or clock is just a convention, so if we interchange X_0 and X_i, $(i = 1, 2, 3)$ the system obviously does not change. In another case, if we interchange two rulers fields, for instance, X and Y, the frame changes from left-hand to right-hand, but we have the state unchanged, $|X_0, X_1, X_2, X_3\rangle = |X_0, X_2, X_1, X_3\rangle$, which implies a reflection symmetry (the parity symmetry) of spacetime coordinates.

A practical example for the identical scalar particles model of a spacetime reference frame is the multi-wire chamber. The beams of the scalar fields used in the model can be considered as free electron fields in the array of multi-wire, which signal the coordinates of an event by an impulse at the output. As it is pointed out, the original electron signal triggered by the event is inescapably quantum mechanical, more precisely, the position of the electron obeys the quantum uncertainty principle.

When the scale of experimental measurement is much larger than a laboratory, the laboratory wall interpretation is no longer useful, since the distance can not be measured directly as in a laboratory. What we could use to measure spacetime coordinates or distance is inevitably only by using the free fields as our rulers and clock. For instance, in the situation of a cosmic observation, the scalar fields used in the model can be instead considered as free photon fields. The information of distance of an event can be extracted from the observations of, for instance, luminosity and frequency/redshift of distant spectral lines.

Here the notion of metric is in general non-trivial in the situation that the measuring of spacetime coordinates is by operationally using the physical fields as rulers and clock. Since a realistic geometric measurement by the free fields in general does not necessarily match the geometry expected beyond the laboratory. For example, a ruler or clock elsewhere measured by a field X_μ or X_0 requires certain technique to compare or synchronize to the laboratory ruler x_a or clock x_0, regardless the laboratory rulers and clock are realistic or extrapolative or complete imaginary in our mind. In a mathematical definition, a vierbein is given by a comparison between the physical coordinates X_μ and the extrapolated absolute laboratory wall frame x_a, $e^\mu_a \equiv \partial X_\mu/\partial x_a$, a metric having dimension $[\text{mass}]^0$ is defined by

$$g^{\mu\nu}(x) \equiv \sum_a e^\mu_a e^\nu_a \equiv \sum_a \frac{\partial X_\mu(x)}{\partial x_a}\frac{\partial X_\nu(x)}{\partial x_a}, \tag{7}$$

in which the extrapolated laboratory wall frame x is assumed homogeneous and flat. The assumption is considered having no impact on the physical result since the physics does not depend on the choice of frame, which can seen

more clearly later. Precisely speaking, we can not align and order the beams of these identical free scalar particles according to the walls and clock of the laboratory in prior, but in general according to the metric $g^{\mu\nu}(x)$ practically measured. In this situation, the Eqs.(4,5) is generalized to

$$S_R[X] = \frac{\lambda}{2}\sum_{\mu,\nu}\int d^4x g_{\mu\nu}\sum_a \frac{\partial X^\mu}{\partial x_a}\frac{\partial X^\nu}{\partial x_a}. \tag{8}$$

Furthermore, without loss of generality, considering the fields are regulated by boundary and initial conditions at the origin $X_\mu(0) = 0$, since here the spacetime reference system is essentially an identical particles system, they have identical wave fronts $\left|\Delta\vec{X}(x)/\Delta X_0(x)\right| = \left|\frac{\vec{X}(x)-\vec{X}(0)}{X_0(x)-X_0(0)}\right| = 1$, where $\vec{X}$ is a vector $\vec{X} = (X_1, X_2, X_3)$. It implies that if an event triggers three vibration configurations, the fourth configuration is physically determined, so we have an additional constraint

$$\sum_{\mu,\nu} g_{\mu\nu}X^\mu X^\nu = \text{const}, \tag{9}$$

in which, due to quantum uncertainties, the constant with dimension $[\text{mass}]^{-2}$ does not simply vanish but is considered related to a length scale of the identical particles. The wave front constraint is necessary to avoid violating the causality, since it keeps the spacetime interval triggered by two events invariant. At this point, the action of the identical scalar particles which describes a spacetime reference frame finally goes to

$$S_R[X] = \int d^dx \left[\frac{\lambda}{2}g_{\mu\nu}\partial_a X^\mu \partial_a X^\nu + \xi\left(g_{\mu\nu}X^\mu X^\nu - \text{const}\right)\right], \tag{10}$$

where the ξ is a Lagrange multiplier that will be functional integrated out and $\partial_a \equiv \frac{\partial}{\partial x_a}$. We have used the Einstein summation convention to sum over indices appearing twice in a single term and neglecting the summation sign, such convention will be used in the rest of the paper.

For historic reasons, the action is known as a non-linear sigma model (NLSM) [16–18] having many applications in particle physics and condensed matter physics. The action Eq.(10) maps a d-dimensional flat absolute parameter background x_a into a D-dimensional target manifold X_μ, which is invariant under local $O(D)$ rotation symmetry, $X_\mu \to X'_\mu = \Lambda_\mu^\nu X_\nu$. The $O(D)$ symmetry can be interpreted as the Lorentz symmetry of the spacetime reference frame, but it is interesting here that the symmetries are induced from the symmetries of the identical particles system which is seen as a model of spacetime reference frame: the reflection symmetry comes from the interchange symmetry of the identical particles, and the rotation symmetry comes from the wave front constraint of the identical particles.

An important point must be emphasized here is that in the rest of the paper the absolute parameter background x_a previously interpreted as an extrapolative absolute laboratory wall frame will have nothing to do with the realistic spacetime. A parameter background is necessary for the theoretical description of a quantum fields theory, but they are not necessarily interpreted as the physical spacetime, the realistic physical spacetime is what we measure from the scalar fields X_μ being a standard reference system. This point is very important in resolving the cosmological constant problem which will be shown in subsection-D of the section III.

Notice that the NLSM classical action is formally proportional to $g_{\mu\nu}g^{\mu\nu} = D$, it is the dimension of the target manifold which is an invariant under the parameter background x-coordinates transformations, so the NLSM is parameter background independent which is the reason we could choose a flat background without loss of any physical generality. Furthermore, the action is also invariant under an arbitrary differentiable X_μ-coordinates transformation of the target manifold. In this sense, a system relative to such spacetime reference frame is connected to a theory of gravity on the target manifold, their relationship will be shown in the next section.

A choice of a reference $\mathcal{H}_R$ is mathematically equivalent to choose a complete set of bases to formulate the whole Hilbert space $\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_R$. More precisely, a choice of a spacetime reference frame explicitly solves the diffeomorphism constraints [8] and the Wheeler-DeWitt equation. The constraints say that physics dose not depend on the choice of reference frame, so if you choose one frame, a physical state must be a summation over all possible choices. In this sense, it again indicates that a physical state in $\mathcal{H}$ is not a simple direct product state, it is an entangled state as Eq.(2) summing over all possible direct product states.

C. Semi-Classical Approximation

In this subsection we will take a first look at the whole system consisting of a to-be-studied system P and a quantum spacetime reference frame R that P is relative to. After the calibration, P and R evolve independently, and we assume

that they do not interact, then the total action is written as

$$S[\varphi, X] = S_P[\varphi] + S_R[X]. \tag{11}$$

Without loss of generality, we consider the to-be-studied system is a scalar fields theory formulated in the same laboratory wall frame as that formulating the spacetime reference system Eq.(10),

$$S_P[\varphi] = \int d^d x \left(\frac{1}{2}\partial_a\varphi\partial_a\varphi - V(\varphi)\right). \tag{12}$$

The total action is then given by

$$S[\varphi, X] = \int d^d x \left[\frac{1}{2}\partial_a\varphi\partial_a\varphi - V(\varphi) + \frac{\lambda}{2}g_{\mu\nu}\partial_a X^\mu \partial_a X^\nu\right], \tag{13}$$

in which we have neglected the constraint term of the NLSM. In a semi-classical approximation, the action can be rewritten as

$$S_{eff}[\varphi] = \int d^4\langle X\rangle \sqrt{\det g}\left[\frac{1}{4}\langle g_{\mu\nu}\rangle \partial_a\langle X^\mu\rangle\partial_a\langle X^\nu\rangle \left(\frac{1}{2}\langle g^{\mu\nu}\rangle \frac{\delta\varphi}{\delta\langle X_\mu\rangle}\frac{\delta\varphi}{\delta\langle X_\nu\rangle} + 2\lambda\right) - V(\varphi)\right], \tag{14}$$

where we have used $g_{\mu\nu}g^{\mu\nu} = D = 4$ and $\sqrt{\det g} = \left\|\frac{\partial x_a}{\partial\langle X^\mu\rangle}\right\|$ is the Jacobian determinant changing the integration variable from x to X. The Jacobian matrix is a square matrix. The property requires the dimension of space x equaling to that of the reference frame fields X, so $d = D = 4$, which is not necessarily true beyond the semi-classical approximation as will be shown in the next section studying its quantum behavior. Since $\langle g^{\mu\nu}\rangle = \partial_a\langle X^\mu\rangle\partial_a\langle X^\nu\rangle$, so $\frac{1}{4}\langle g_{\mu\nu}\rangle\partial_a\langle X^\mu\rangle\partial_a\langle X^\nu\rangle = 1$. We obtain

$$S_{eff}[\varphi] = \int d^4\langle X\rangle\sqrt{\det g}\left[\frac{1}{2}\langle g^{\mu\nu}\rangle\frac{\delta\varphi}{\delta\langle X_\mu\rangle}\frac{\delta\varphi}{\delta\langle X_\nu\rangle} - V(\varphi) + 2\lambda\right]. \tag{15}$$

The effective action obtained from the semi-classical approximation has a straightforward interpretation: the to-be-studied quantum fields system of φ is relative to the semi-classically treated spacetime reference frame X_μ. The equation Eq.(15) is similar with Eq.(12) up to an unimportant constant, just formally the derivative $\frac{\partial}{\partial x}$ is replaced by a functional derivative $\frac{\delta}{\delta\langle X\rangle}$, and the function $\varphi(x)$ with respect to x is replaced by a functional $\varphi[X]$ with respect to X.

Furthermore, although parameter background space x is flat, when the target manifold is curved, the effective action includes such a theory that field or even quantized field φ is in a curved spacetime. We have proved the equivalence between the semi-classical theory of the spacetime reference frame and the theories of (quantum) fields in curved spacetime. Thus it is reasonable to expect that the theory could recover the existing results of quantum fields theories in curved spacetime, for instance the Hawking's radiation of a black hole.

However, only a semi-classical treatment of the spacetime reference frame is not enough, where the spacetime is still fix, the quantum dynamics of the spacetime reference frame must be considered. We will show in the next section that an effective Einstein's theory of gravity and a correct cosmological constant naturally arise from the quantum behavior of the spacetime reference frame.

III. QUANTUM BEHAVIOR

In this section, we will study the quantum dynamics of the spacetime reference frame defined in previous section. Let us recall the NLSM in Eq.(10), which maps a d dimensional flat homogeneous parameter background $x_a, (a = 0, 1, 2, ..., d-1)$ into a D-dimensional target manifold $X^\mu, (\mu = 0, 1, 2, ..., D-1)$ with $O(D)$ symmetry. Here we interpret the target manifold coordinates X^μ as scalar fields describing the coordinates of the realistic spacetime, so $D \equiv 4$ is considered fixed. However, we must not fix d from the beginning since it is known that the dimension of the parameter background d runs in a renormalization flow, the discrepancy is known as anomalous dimension. Therefore here the parameter background space has nothing to do with our realistic spacetime any more in the theory, and d is considered varying with the scale of renormalization. The action of the NLSM is

$$S_{NLSM} = \frac{\lambda}{2}\int d^d x g_{\mu\nu}[X]\partial_a X^\mu\partial_a X^\nu, \tag{16}$$

where $g_{\mu\nu}[X]$ is a positive dimensionless metric of the target manifold, and since coordinates x_a and X^μ have dimensions $[\text{mass}]^{-1}$, the constant λ has dimension $[\text{mass}]^d$.

A. Renormalization

Most of the quantum behaviors of a system encode in its renormalization. Here we are interested in the dimensionless field renormalization function $Z(k) = 1 + \delta_Z(k)$, which scales each field as,

$$X^\mu \to X^\mu_{re} = Z(k) X^\mu. \tag{17}$$

The k, with dimension $[\text{mass}]^1$, introduced by hand measures a cutoff of the Fourier component of the dynamical spacetime fields X^μ. So here k replaces the absolute parameter background x, playing the role of an evolution parameter. Then the action in a "Wilsonian" sense is effective defined at the cutoff k,

$$S_k = \frac{\lambda}{2} \int d^d x Z g_{\mu\nu} \partial_a X^\mu \partial_a X^\nu. \tag{18}$$

The renormalization function Z can also be absorbed into the constant λ or $g_{\mu\nu}$ while X^μ are fixed, which has completely different physical interpretation but the mathematics is the same.

Recall that, in the semi-classical approximation, the dimensions of the parameter background and that of the target manifold are identical, i.e. $d = D \equiv 4$. Although it seems that the NLSM in $d = 4$ is perturbative non-renormalizable by means of power counting, but at non-perturbative level, it is shown [19] that a $d = 4$ NLSM has a non-trivial (non-Gaussian) UV fixed point (i.e. $k \to \infty$). In this sense, this theory of quantum spacetime is "asymptotically safe"[20], which will be clearly seen later by noting that d runs to 2 at $k \to \infty$. Therefore, the fields X^μ and λ in Eq.(16) can be seen as bare values defined at the UV fixed point, at which the renormalization condition is written as

$$\lim_{k\to\infty} Z(k) = 1, \quad \lim_{k\to\infty} \delta_Z(k) = 0. \tag{19}$$

In order to find the physics at IR, the question we want to ask is simple: from this initial renormalization condition at UV, what value does it take when the renormalization group flows to IR ($k \to 0$)?

The answer to the question for the $d = 4$ NLSM has been studied in literature by perturbation theory when the value of λ is large [19] (the coupling of the NLSM being the inverse of λ). Fortunately in our theory the bare value λ defined at UV is indeed large. It is of order $\lambda \sim \mathcal{O}(R k_{UV}^{d-2})$ [19], where R is the Ricci scalar curvature of the target manifold and k_{UV} is a constant relating to a UV scale much larger than the scale k we are interested in, i.e. $k_{UV} \gg k$.

In this situation, a large λ corresponds to a small coupling, the perturbation calculation of δ_Z is reliable, which at one loop is given by

$$\delta_Z(k) = \frac{1}{2} \frac{R}{\lambda D} \int_{0 \le |p| < k} \frac{d^d p}{(2\pi)^d} \frac{i}{p^2} = \frac{1}{(4\pi)^{d/2} \Gamma(\frac{d}{2})(d-2)} \frac{R}{\lambda D} k^{d-2}, \tag{20}$$

where R is a positive Ricci scalar curvature with dimension $[\text{mass}]^2$. By using $d = 4 + \epsilon$ expansion and the minimal subtraction scheme, we obtain a regularized function

$$\delta_Z(k) = C_d \frac{R}{\lambda D} k^{d-2} + C, \tag{21}$$

where $C_d^{-1} = (4\pi)^{d/2} \Gamma(d/2 + 1)$ and C represents an integral constant taking the value $\delta_Z(0)$ to be determined by the initial renormalization condition Eq.(19). Thus for $d = 4$ the renormalization function in the IR region behaves like

$$\delta_Z(k) = \frac{1}{128\pi^2} \frac{R}{\lambda} k^2 + C, \quad (\text{small } k). \tag{22}$$

To determine the integral constant C we need to apply the initial renormalization condition defined at UV. However, the function Eq.(22) is only valid at IR, obvious it diverges in the limit $k \to \infty$. To a large extent, the difficulty can be attributed to the fact that such behavior of δ_Z will completely change in the UV region as a consequence of the running of the effective dimension of the system. As is discussed in literature, the anomalous dimension is equal to $\eta = d - 2$ at the non-trivial UV fixed point [19], so the effective dimension near the UV fixed point tends to $d_{eff} = d - \eta = 2$, but rather 4. In this case, the power of k is zero in Eq.(20), so δ_Z is expected to behave as $\log k$ in the UV region. Therefore, one could see that the dimension d and the behavior of $\delta_Z(k)$ are very different at small k and large k region,

$$d = \begin{cases} 4 & (k \to 0) \\ 2 & (k \to \infty) \end{cases}, \qquad \delta_Z \sim \begin{cases} k^2 & (\text{small } k) \\ \log k & (\text{large } k) \end{cases}. \tag{23}$$

We can see that the growth rate $\partial\delta_Z/\partial k$ is always positive, which means δ_Z grows and the coupling decreases with the increasing of k. However, it does not mean the theory is asymptotically free, it is in fact "asymptotically safe". The reason is transparent, although at IR δ_Z grows as $\sim k^2$, the rate slows down to $\sim (d-2)k^{d-2}$ in the UV region, and finally the growth rate vanishes as $d \to 2$ and the coupling approaches to a finite value at the UV fixed point.

In order to explicitly calculate the renormalization function in the UV region, a viable approach is to use the free fields or linear approximation, namely let $g_{\mu\nu} \approx \eta_{\mu\nu}$, and ignore the non-linear interactions in NLSM which are seen as small corrections. The using of such approximation to study its quantum behavior at UV is reasonable, since as discussed previously the coupling of the theory tends to a small value at UV and it is "asymptotically safe", the result in the free fields limit must dominate. In this approximation, the NLSM becomes quadratic in X, each component decouples to other fields and enters independently in Eq.(10). Notice that the λ has dimension $[\text{mass}]^2$ when $d=2$, so we could rescale the fields X to enforce a dimensionless constraint $\frac{1}{2}\lambda X_\mu X^\mu = 1$. If we work in the $d=2$ Euclidean parameter background, the partition function can be written as

$$\mathcal{Z} = \int \mathcal{D}\xi\mathcal{D}X \exp\left[-\int d^2x \left(Z\frac{1}{2}\lambda\left(\partial X\right)^2 + Z\frac{1}{2}\xi(\lambda X^2 - 1)\right)\right]. \tag{24}$$

Since here the action is Gaussian, integrating over X, we have

$$\mathcal{Z} = \mathcal{N}\int \mathcal{D}\xi \exp\left[-\frac{D}{2}\mathrm{tr}\log(-\partial^2 + \xi) + \frac{1}{2}\int d^2x Z\xi\right]. \tag{25}$$

The renormalization function Z plays the role of a response to the external source field ξ , so let us compute the variation of the exponent with respect to ξ and finally taking the sourceless limit $\xi \to 0$,

$$Z = D\,\frac{\delta}{\delta\xi}\mathrm{tr}\log(-\partial^2 + \xi)\bigg|_{\xi\to 0}. \tag{26}$$

Close to $d=2$ it leads to

$$Z(k) = D\int_{M\leq|p|<k} \frac{d^2p}{(2\pi)^2}\frac{i}{p^2}. \tag{27}$$

in which M is an arbitrary renormalization scale. We obtain the integral at the UV cut off k,

$$\delta_Z(k) = Z(k) - 1 = \frac{D}{2\pi}\log\frac{k}{M} + C, \quad (\text{large } k). \tag{28}$$

which is a counterpart of Eq.(22) at asymptotic large k.

In order to determine the k-independent constant C, we make use of the continuity and universality of the scaling dimension γ function in renormalization. At small k where $d_{eff} = 4$, by using Eq.(22) the γ function is given by

$$\gamma(k) = \frac{1}{2}k\frac{\partial\delta_Z}{\partial k} = \frac{1}{128\pi^2}\frac{R}{\lambda}k^2, \quad (\text{small } k), \tag{29}$$

while at large k where $d_{eff} \to 2$, by using Eq.(28) it changes from the quadratic behavior to a universal critical scaling dimension

$$\gamma_c = \gamma(k\to\infty) = \frac{1}{2}k\frac{\partial\delta_Z}{\partial k} = \frac{D}{4\pi}, \quad (\text{large } k). \tag{30}$$

Since γ is a smooth function of k and, in the asymptotic UV region, it continuously approaches to the universal value independent to the cutoff k, they must be identical at certain large k_i,

$$\gamma(k_i) = \frac{1}{128\pi^2}\frac{R}{\lambda}k_i^2 = \frac{D}{4\pi}. \tag{31}$$

At such large scale k_i where the γ function becomes universal and hence the scaling of the system starts becoming critical, it is safe to consider the initial renormalization condition Eq.(19) starts applying. Substituting the Eq.(31) into Eq.(22) we have

$$\delta_Z(k_i) = \frac{D}{4\pi} + C = 0, \tag{32}$$

which immediately demands a universal value of C relating to the universal γ_c at UV, by using $D \equiv 4$,

$$C = -\gamma_c = -\frac{1}{\pi}. \tag{33}$$

Despite the above result is evaluated under the one-loop approximation, more precisely speaking, the result actually can be seen reliable and solid. Since in the vicinity of the UV fixed point, the system becomes nearly critical, the renormalization function $Z(k)$ is nothing but a simple power law

$$Z(k) \sim \left(\frac{k^2}{M^2}\right)^{\gamma(k)+C}, \quad \text{(large } k\text{)}.$$

Therefore, by applying the initial renormalization condition, $Z(k \to \infty) = 1$, the critical exponent must be zero, so the constant C can again be universally determined by $C = -\gamma_c$ independent to the choice of the cutoff.

At this moment, we arrive at an asymptotic function δ_Z in the IR region, which is able to extrapolatively satisfy the initial renormalization condition at UV,

$$\delta_Z(k) = -\frac{1}{\pi} + \frac{1}{128\pi^2}\frac{R}{\lambda}k^2, \quad \text{(small } k\text{)}. \tag{34}$$

For this formula, we should not incorrectly consider that k must stop at a particular point k_c to enforce the renormalization condition ($\delta_Z(k_c) = 0$). We must emphasis that k will not stop at k_c, it could continuously go to infinity, since at large k the behavior of δ_Z changes as the effective dimension approaches 2 but rather 4, and δ_Z increases as $\log k$ but rather k^2. We know that the NLSM near $d = 2$ becomes perturbative renormalizable [21, 22] which is a positive feature for a good behavior of our model at UV. And finally the increasing rate slows down and stop at the UV fixed point, where the δ_Z vanishes as the renormalization condition enforces, leaving the finite bare λ. In this sense, the theory truly has a non-trivial UV fixed point, where the theory is well-defined at quantum level with finitely many relevant bare inputs.

On the other hand, note that the critical scaling dimension at UV affects the IR behavior non-trivially, as $k \to 0$, $Z(k)$ changes from unity at UV limit to $1 - \gamma_c$ at IR limit. As a consequence, the theory also has a non-trivial IR fixed point, the λ is renormalized to be a finite value at IR,

$$\lambda_{IR} = \left(1 - \frac{1}{\pi}\right)\lambda \approx 0.68\lambda. \tag{35}$$

Note that the λ_{IR} is of the same order of the UV bare value λ, so the perturbation technic is not only valid for UV but also IR. In this sense, we consider the above results done at small k are also reliable. As we will discuss later, the non-trivial IR fixed point is very crucial for understanding the observational large distance cosmology, such as accelerating expansion or cosmological constant.

B. Effective Action: Emergent Cosmological Constant

In this subsection, we study the behavior of the system in the IR region. Substituting the Eq.(34) into Eq.(18), taking $d = 4$, one get the effective action in the IR region,

$$S_k = \frac{1}{2}\int d^4x \left(\lambda - \frac{1}{\pi}\lambda + \frac{1}{128\pi^2}Rk^2\right) g_{\mu\nu}\partial_a X^\mu \partial_a X^\nu, \tag{36}$$

in which the parameter λ is a x-independent constant and hence can be taken into the integral. By definition λ is a k-independent UV bare value, and relates to a UV scale k_{UV} and a fixed point value of scalar curvature [19]. Perturbative calculation shows that it is of order $\lambda \sim \mathcal{O}(\langle R\rangle k_{UV}^2)$, where $\langle R\rangle$ is a volume averaged constant scalar curvature, $\langle R\rangle = \int R dv / \int dv$, whose value can be obtained from a Ricci-type flow equation. The precise value of λ can be determined by re-parameterizing it as $\lambda = \frac{\mathfrak{a}\langle R\rangle}{32\pi G}$ and finding the numerical factor $\mathfrak{a}$, where G is the Newton's constant playing the role of the k_{UV}^{-2}. We rewrite the action as

$$S_k = \frac{1}{4}\mathfrak{a}\int d^4x \left(\frac{\langle R\rangle}{16\pi G} - \frac{2}{\pi}\lambda + \frac{1}{64\pi^2\mathfrak{a}}Rk^2\right) g_{\mu\nu}\partial_a X^\mu \partial_a X^\nu. \tag{37}$$

The term $\langle R\rangle$ is a numerical constant representing a low energy mean curvature background, and the k-dependent renormalization correction term describes the curvature fluctuation about the background. A correct effective action of gravity at well-tested distance scale where the cosmological constant plays little role, should be the standard Einstein-Hilbert action. In other words, at that scale the $-(2/\pi)\lambda$ term is largely canceled by the renormalization correction term, leaving the effective action at the scale appear as the Einstein-Hilbert action. By comparison of the prefactor with that in the semi-classical action Eq.(14), one has $\mathfrak{a}=1$. It is worth mentioning that the separate interpretation of λ and $-(1/\pi)\lambda$ is by convention and just an artifact of the particular choice of the renormalization scale namely the Einstein-Hilbert truncation, in fact they are indistinguishable because of the equivalence principle, we will come back to this point in the subsection D.

Therefore, substituting the effective action of reference frame into the Eq.(11) and using the semi-classical approximation to treat X_μ, we obtain a total effective action formulated by using the internal and physical coordinates X_μ,

$$S_{eff}[\varphi,\langle g_{\mu\nu}\rangle]=\int d^4\langle X\rangle\sqrt{\det g}\left[\frac{1}{2}\langle g^{\mu\nu}\rangle\frac{\delta\varphi}{\delta\langle X_\mu\rangle}\frac{\delta\varphi}{\delta\langle X_\nu\rangle}-V(\varphi)+\frac{\langle R\rangle}{16\pi G}-\frac{2}{\pi}\lambda+\frac{1}{64\pi^2}Rk^2\right]. \tag{38}$$

The first two terms in the bracket are the ordinary matter term, the third term by convention can be interpreted as the low energy Einstein-Hilbert term. The fourth term $(2/\pi)\lambda$ is a positive constant being of order $\langle R\rangle k_{UV}^2$, which consequently is interpreted as the effective cosmological constant corresponding to an energy density

$$\rho_\Lambda=\frac{2}{\pi}\lambda. \tag{39}$$

Note that the critical density is defined by $\rho_c=\frac{3H^2}{8\pi G}$, where H is the Hubble parameter. The value near current epoch $H_0=H|_{z\sim 0}$ then gives $\rho_c|_{z\sim 0}=\frac{3H_0^2}{8\pi G}$, which is a density averaged by a volume with respect to rulers and clocks in our near current epoch. By using the relation between the mean Ricci curvature and the curvature radius, $\langle R\rangle\equiv D(D-1)r^{-2}=12r^{-2}$, where the curvature radius r is given by the current Hubble parameter $r=H_0^{-1}$, so one find that the critical density is just the UV fixed point value of λ,

$$\rho_c|_{z\sim 0}=\lambda=\frac{\langle R\rangle}{32\pi G}. \tag{40}$$

Then we have $\rho_\Lambda\doteq(2/\pi)\ \rho_c|_{z\sim 0}$, where "$\doteq$" stands for the neglecting of the renormalization correction. It is an interesting result that the dark energy is equal to $2/\pi$ times the "current" critical density,

$$\Omega_\Lambda|_{z\sim 0}=\frac{2}{\pi}\approx 0.64, \tag{41}$$

which agrees with the observations well. The word "current" means that the density is averaged by the volume relative to the scales of rulers and clocks near $z\sim 0$. Certainly, $\rho_\Lambda\doteq(2/\pi)\lambda$ and $(2/\pi)\ \rho_c|_{z\sim 0}$ are just equal in values, they are not really identical. $\rho_\Lambda\doteq(2/\pi)\lambda\propto\langle R\rangle$ is a scalar, it gives rise to a full stress tensor $T_{\mu\nu}=\rho_\Lambda g_{\mu\nu}$, for this reason, its equation of state is exactly $w=-1$, in contrast, ρ_c is just the 00 component of a stress tensor, so they behave differently under spacetime coordinates transformation. It is easy to see that $\rho_\Lambda\doteq(2/\pi)\lambda\propto\langle R\rangle$ is invariant with respect to the physical time, in this sense it is a constant, but $(2/\pi)\rho_c$ varies with the redshift, they are just equal in values near $z\sim 0$.

In the framework only the concepts defined in a relative way, but rather absolute, are observable. A physical observable is redshift z, while the absolute global age of the universe is essentially unobservable in our theory. In this sense, discussions about the universe evolution should not base on the global age but the redshift. In other words, the quantities being functions of the global age, e.g. $H(t)$, $\rho_i(t)$, $\Omega_i(t)$ should be replaced by some more physical ones being functions of the redshift: $H(z)$, $\rho_i(z)$, $\Omega_i(z)$. It is no problem that the near current epoch $z\sim 0$ always exist in every epoch of the expansion history of the universe, leading to the consequence that an observer at his/her near "current" epoch "always" find $\Omega_\Lambda|_{z\sim 0}\approx 0.64$ no matter what is the absolute epoch he/she lives. In this sense, ρ_Λ is "always" comparable with $\rho_c|_{z\sim 0}$ seeing by observers at his/her epoch. The word "always" is about the relational redshift but rather the absolute history or age, meaning that the value of Ω_Λ is fixed with respect to the observers at $z\sim 0$, it does not mean ρ_Λ and ρ_c behave in the same way in the expansion history of the universe. As we know, they behave differently about the redshift, $\rho_c(z)$ increases with z, while ρ_Λ does not change, resulting to that $\Omega_\Lambda(z)=\rho_\Lambda/\rho_c(z)$ decreases as redshift increases. The framework gives a non-dynamical explanation to the coincidence problem, namely why the dark energy is comparable with the critical density now? The key to readily understand the coincidence is to use the relational observable (such as redshift) independent to any absolute scale of the universe, avoiding using the quantity such as the global age of the universe defined by an absolute scale look from its outside. We will also come back to the coincidence problem in discussing the distance-redshift relation in the next subsection.

C. Physical Interpretation: Spacetime Uncertainties at Cosmic Distance

We have seen that the renormalization function δ_Z or Z is powerful, the cosmological constant explicitly emerged from it. In this subsection, we will discuss its physical interpretation.

The previous renormalization results can also be understood by the mean field method. In such language, if we use the differences between the fields values to a given boundary condition (e.g. $X_\mu(0)$) to be dynamical variables, i.e. $\Delta X_\mu(x) = X_\mu(x) - X_\mu(0)$, the difference of renormalized physical field ΔX_μ^{re} can be expanded around the mean value $\langle \Delta X_\mu \rangle$ by a quantum fluctuation δX_μ, in k-space it is formulated as

$$\Delta X_\mu^{re}(k) = \langle \Delta X_\mu \rangle + \delta X_\mu(k). \tag{42}$$

By using this relation and Eqs.(17,18), considering that the vacuum expectation value of the fluctuation vanishes $\langle \delta X_\mu \rangle = 0$, we find that $\delta_Z(k)$ is nothing but a measure of a ratio of each variance to the mean-squared of its spacetime coordinate difference,

$$\delta_Z(k) = Z(k) - 1 = \frac{(i)^2}{2} \frac{\langle \delta X_\mu^2(k) \rangle}{\langle \Delta X_\mu \rangle^2}, \tag{43}$$

in which the factor $(i)^2 = -1$ is given by the pre-factor $(i/\hbar)^2$ in the standard perturbation series and the natural unit $\hbar = 1$ is taken. Recall the result Eq.(34), we have

$$\frac{\langle \delta X_\mu^2(k) \rangle}{\langle \Delta X_\mu \rangle^2} = -2\delta_Z = \frac{2}{\pi}\left(1 - \frac{1}{4}k^2 G\right), \quad \text{(small } k\text{)}. \tag{44}$$

This formula indicates an inescapable and universal quantum limitation to the spacetime accuracy at IR. One can not have rulers and clocks precisely measured and synchronized across spacetime. In the IR limit, or equivalently, at cosmic scale, the ratio of the variance $\langle \delta X_\mu^2 \rangle$ to the mean-squared of spacetime distance $\langle \Delta X_\mu \rangle^2$ is universal,

$$\lim_{k \to 0} \frac{\langle \delta X_\mu^2 \rangle}{\langle \Delta X_\mu \rangle^2} = \frac{2}{\pi}. \tag{45}$$

The universality of the ratio is closely related to the universality of the critical dimension γ_c in Eq.(33). In the next subsection, we will see that the universality of the ratio is also a natural consequence of a generalized equivalence principle.

The relation Eq.(45) for the variance $\langle \delta X_\mu^2 \rangle$ is valid at extreme IR or cosmic distance $\langle \Delta X_\mu \rangle^2$, which can be seen as an inherent cosmic variance of a quantum measurement at cosmic distance. It generalizes the result of refs.[4, 5] by putting the space and time on an equal footing. In those papers, only the uncertainty of a physical clock time field X_0 is considered, nevertheless the spatial coordinates are just treated as parameter background for the sake of simplicity. The difference between them is as follow. For the result in refs.[4, 5], the clock time variance grows linearly with the spatial distance and the proportional coefficient is the inverse of the spatial volume cutoff. However, in this paper the clock variance instead grows quadratically but linearly. Since the space and time is considered symmetric and isotropic, the clock variance can be interpreted as being proportional to the squared spatial distance. The reason for the different power dependence is due to the fact that, in standard quantum mechanics, it is of first order in the derivative with respect to an evolution parameter, for instance the derivative with respect to the Schrodinger's time or the renormalization scale $\log k$, but in a theory that space and time are put on an equal footing, for instance a relativistic theory, the orders in the derivatives with respect to spatial and temporal distance are the same, for bosonic degrees of freedom, they are both of second order.

For the reason that time or frequency can be conveniently measured by redshifts of distant spectral lines in cosmic observations, they could be used for good clocks far away testing the effect at cosmic distance. The clock time uncertainty can also be interpreted in terms of a redshift uncertainty or broadening. Although many physics affect the redshift broadening such as thermal fluctuation, a prediction may be testable: the ratio of the inherent quantum variance of redshift to its mean-squared equals

$$\lim_{k \to 0} \frac{\langle \delta z^2 \rangle}{\langle z \rangle^2} = \frac{2}{\pi}, \tag{46}$$

which is derived from Eq.(45) by using $\left\langle \delta z^2 \right\rangle = \left\langle \delta X_\mu^2 \right\rangle / \left\langle X_\mu(0) \right\rangle^2$ and $\langle z \rangle = \langle \Delta X_\mu \rangle / \langle X_\mu(0) \rangle$. The result is also deduced in ref.[5]. The $\langle \delta z^2 \rangle$ does not change the mean value $\langle z \rangle$, but shifts the mean-squared

$$\langle z^2 \rangle = \langle z \rangle^2 + \langle \delta z^2 \rangle \overset{k \to 0}{=} \left(1 + \frac{2}{\pi}\right) \langle z \rangle^2. \tag{47}$$

Therefore, the distance-redshift relation $D(z)$ is modified at order $\mathcal{O}(z^2)$ by this effect [5]. The first order $\mathcal{O}(z)$ term of $D(z)$ relates to the expansion rate of the universe, and the second order $\mathcal{O}(z^2)$ term relates to an accelerating or decelerating of the expansion. More precisely, the universal ratio Eq.(46) contributes an additional deceleration parameter $q_0 = -2/\pi$, which is redshift independent and uniform, to the distance-redshift relation besides other components of the universe. It makes a departure to the Hubble's law which is more significant at high redshift. Expanding the expectation value of the luminosity distance $\langle D(z)\rangle$ in powers of redshift to the second order [23], we have

$$\langle D(z)\rangle = \frac{1}{H_0}\left[\langle z\rangle + \frac{1}{2}\left(1+\frac{2}{\pi}+...\right)\langle z\rangle^2 + \mathcal{O}(\langle z\rangle^3)\right], \tag{48}$$

where the additional positive constant $2/\pi$ plays the role of the percentage of dark energy Ω_Λ, the ... in the parenthesis represents the deceleration parameter coming from other components of the universe such as ordinary matter given by $-\Omega_m(1+z)^3$. H_0 is the Hubble constant measured at $z \sim 0$. The formula does not involve any absolute age of the universe, so for any observer, it is always valid no matter when the observer lives in the expansion history of the universe, the universe is always seen to become accelerating at near current epoch $z \approx 0.3$ at which the dark energy and the matter become comparable, i.e. $q_0 = \Omega_m(1+z)^3 - \Omega_\Lambda = 0$. This fact also demonstrates the coincidence. The $\left\langle \delta z^2 \right\rangle$ correction to the distance-redshift relation makes it become anomalous at high redshift, which is observed from high redshift supernovas being the first indication of the accelerating expansion of the universe [24, 25].

D. A Generalized Equivalence Principle

In previous sections, we have shown that a correct value of cosmological constant and an Einstein-Hilbert term emerge in the IR region of the quantum dynamics of spacetime reference frame. From the dynamics of quantum spacetime reference frame to the concept of gravity, a further assumption is required: the equivalence principle. The principle gives a spacetime interpretation of gravity in classical general relativity, which is well established and tested in classical physics. However, the principle puzzles physicist when the quantum effects are seriously taken into account. Since the zero-point quantum fluctuation seems real (e.g. contributing to the Lamb shift), but why these large amount of energies of vacuum do not gravitate as the equivalence principle asserts, which is a main puzzle of the cosmological constant problem.

In fact the Lamb shift gravitates normally [26, 27], and there is no hint to assume that the energies coming from classical and quantum contributions produce different gravitational effects. In fact, only when the equivalence principle is valid, the accelerating expansion of the universe revealed by Eq.(48) is equivalent to the existence of a "dark energy" or a positive non-vanishing cosmological constant revealed by Eq.(38), so we have already felt a rough indication that in our framework the equivalence principle could exactly hold. Furthermore, an elegant and economic assumption is also to maintain the spirit of relativity and hence the equivalence principle, so that gravity is just simply a relative phenomena, and there is nothing else more than that even at quantum level. In this sense, the equivalence principle must be generalized to the quantum level to resolve the cosmological constant problem.

In this paper, we argue that the equivalence principle is also valid for all quantum phenomenon as it applies to the classical phenomenon. As the generalized equivalence principle asserts, all kinds of energies including the quantum fluctuations gravitate. The generalized equivalence principle implies that all kinds of apparent curving of spacetime including those coming from quantum fluctuations or quantum uncertainties of spacetime are equivalent to gravitation, more precisely, the quantum uncertainties of spacetime deduced in this paper are equivalent to the accelerating expansion of the universe. Not only one can not distinguish gravitation from acceleration according to the classical equivalence principle, but also unable to distinguish gravitation from quantum spacetime intrinsic uncertainty/fluctuation. Therefore, the Ricci curvature in the effective action Eq.(38) can precisely be interpreted as gravity. The accuracy of the generalized equivalence principle can be demonstrated by the universality of the ratio Eq.(46) which is independent to the energies of spectral lines. In other words, all spectral lines taking different energies uniformly "free-fall", it is not merely a particular property of the spectral lines, it is a universal property of the spacetime itself, so a universal accelerating expansion inevitably appears. The uncertainty/fluctuation of spacetime Eq.(45) or redshift Eq.(46) on the one hand can be interpreted as that the objects (spectral lines) are uniformly accelerating, or equivalently on the other hand, the spacetime is curved by the quantum fluctuation energy density $(2/\pi)\lambda$ in Eq.(38) which is seen as a repulsive gravitational force.

To understand why the generalized equivalence principle resolves the notorious problem of zero-point vacuum energies $\sum_k \frac{1}{2}\omega_k$ predicted in textbook quantum fields theory, note that such zero-point vacuum energies are not involved in our effective theory Eq.(38). The effective vacuum energies density or the cosmological constant as a source of gravity in Eq.(38) comes from the two-point function $\langle \delta X_\mu \delta X_\nu \rangle \neq 0$, while the vacuum expectation value vanishes, $\langle \delta X_\mu \rangle = 0$. In other words, the energies of quantum fluctuations of spacetime relating to two states are

the leading contribution to the vacuum energies and gravitational effects [28, 29], which do obey the generalized equivalence principle. However, the notorious zero-point vacuum energies relating to one state have no gravitational effects. Since quantum fluctuation of a physical reference frame is inevitable, fundamentally speaking, the absolute rest frame can not be precisely realized at quantum level. For this reason, the zero-point vacuum energies which make sense only with respect to a perfect classical absolute rest frame in textbook quantum mechanics are completely unphysical and unobservable. This property resolves the first part of the cosmological constant problem, namely, why the zero-point vacuum energies do not gravitate. The physical energies which gravitate normally are those make sense with respect to the quantum spacetime reference frame that is also subject to quantum fluctuation. As an application, the dark energy from the vacuum fluctuation $\langle \delta X_\mu \delta X_\nu \rangle$, which is relative to each other, gravitate normally as the generalized equivalence principle asserts. This property solves the second part of the cosmological constant problem, namely, why the cosmological constant is so small.

IV. SUMMARY AND CONCLUSIONS

To solve the cosmological constant problem, the quantum dynamics and effects of a quantum clock can not be neglected. In this paper, acting on the spirit of treating space and time on an equal footing, we generalize the quantum clock to the quantum spacetime reference frame, via a physical realization of a reference system by quantum rulers and clocks. It is in this sense we have a "quantum spacetime" obeying both quantum mechanics and general relativity.

In order to accommodate quantum mechanics to general relativity, the textbook quantum mechanics must be generalized. General relativity is general covariant or observer independent, a physical quantum state satisfying this property is in general an entangled state entangling a to-be-studied quantum system with a quantum measuring device. It is necessary to have a relational interpretation to the state, since the to-be-studied quantum system makes sense only relative to the quantum measuring device. Entangled state solves the diffeomorphism constraints and the Wheeler-DeWitt equation, which plays a more fundamental role than the textbook Schrodinger equation. Since clock time is inescapably subject to quantum fluctuation, the Schrodinger equation using the parameter time is just an approximation. The cosmological constant problem is an indication of going beyond the Schrodinger equation where the quantum fluctuation of time is inescapable and must not be ignored.

Omitting the internal degrees of freedom of the physical rulers and clocks, such as spins, considering only their metric properties, the spacetime reference frame is described by a bosonic non-linear sigma model. In a semi-classical treatment of the spacetime reference frame, we recover the existing theories: (quantum) fields theories in curved spacetime. In a complete quantum treatment, we studied its normalization behavior under approximations. The theory has a non-trivial UV fixed point, namely it is asymptotically safe, and hence the notion of spacetime still makes sense even at UV. We get three surprising results from the theory. The first, and most remarkable, result is that a cosmological constant appears which again gives not only a correct order but also a percentage $\Omega_\Lambda = 2/\pi \approx 0.64$ very close to current observational value. The second result is that the quantum dynamics of the quantum spacetime automatically contains an Einstein-Hilbert term, and hence automatically incorporates a theory of gravity under the assumption of the validity of equivalence principle at quantum level. The third result says that the spacetime are inescapably subject to quantum uncertainties, one can not have rulers and clocks perfectly measured and synchronized across spacetime. The ratio of the variance to the mean-squared of the spacetime distance tends to a universal constant $\langle \delta X_\mu^2 \rangle / \langle \Delta X_\mu \rangle^2 = 2/\pi$ in the extreme IR region of theory. We also argue that this effect is testable by observing a linear dependence between the variance and mean-squared of redshifts from distant spectral lines. The proportionality is $\mathcal{O}(1)$ and expected to be identical to the percentage of the dark energy Ω_Λ. These results strongly support the argument that the equivalence principle still holds at quantum level. It is in this sense we propose a possible "quantum theory of gravity".

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[1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[2] R. Bousso, General Relativity and Gravitation **40**, 607 (2008), 0708.4231.

[3] J. Martin, Comptes Rendus Physique **13**, 566 (2012), 1205.3365.
[4] M. J. Luo, Nuclear Physics B **884**, 344 (2014), 1312.2759.
[5] M. J. Luo, Journal of High Energy Physics **6**, 63 (2015), 1401.2488.
[6] Planck Collaboration, P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday, et al., ArXiv e-prints (2013), 1303.5062.
[7] Y. Aharonov and T. Kaufherr, Phys. Rev. D **30**, 368 (1984).
[8] C. Rovelli, Classical and Quantum Gravity **8**, 317 (1991).
[9] M. Dickson, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **35**, 195 (2004).
[10] R. M. Angelo, N. Brunner, S. Popescu, A. J. Short, and P. Skrzypczyk, Journal of Physics A: Mathematical and Theoretical **44**, 145304 (2011).
[11] D. N. Page and W. K. Wootters, Phys. Rev. D **27**, 2885 (1983).
[12] E. Moreva, G. Brida, M. Gramegna, V. Giovannetti, L. Maccone, and M. Genovese, Phys. Rev. A **89**, 052122 (2014), 1310.4691.
[13] C. Rovelli, International Journal of Theoretical Physics **35**, 1637 (1996), quant-ph/9609002.
[14] C. Rovelli, *Quantum gravity* (Cambridge University Press, 2004).
[15] N. G. Phillips and B. Hu, Physical Review D **62**, 084017 (2000).
[16] M. Gell-Mann and M. Lévy, Il Nuovo Cimento **16**, 705 (1960).
[17] S. V. Ketov, *Quantum non-linear sigma-models: from quantum field theory to supersymmetry, conformal field theory, black holes and strings* (Springer Science & Business Media, 2000).
[18] J. Zinn-Justin, *Quantum field theory and critical phenomena* (Oxford University Press, Oxford, 2002).
[19] A. Codello and R. Percacci, Physics Letters B **672**, 280 (2009).
[20] S. Weinberg, in *S. W. Hawking and W. Israel (Eds), General relativity: An Einstein centenary survey* (Cambridge University Press, 1979).
[21] E. Brezin, J. Zinn-Justin, and J. Le Guillou, Physical Review D **14**, 2615 (1976).
[22] D. Friedan, Physical Review Letters **45**, 1057 (1980).
[23] S. Weinberg, *Cosmology* (Oxford University Press, Oxford, 2008).
[24] A. G. Riess et al. (Supernova Search Team), Astron.J. **116**, 1009 (1998), astro-ph/9805201.
[25] S. Perlmutter et al. (Supernova Cosmology Project), Astrophys.J. **517**, 565 (1999), astro-ph/9812133.
[26] J. Polchinski, ArXiv High Energy Physics - Theory e-prints (2006), hep-th/0603249.
[27] E. Massó, Physics Letters B **679**, 433 (2009), 0902.4318.
[28] T. Padmanabhan and T. Singh, Classical and Quantum Gravity **4**, 1397 (1987).
[29] T. Padmanabhan, Classical and Quantum Gravity **22**, L107 (2005), hep-th/0406060.